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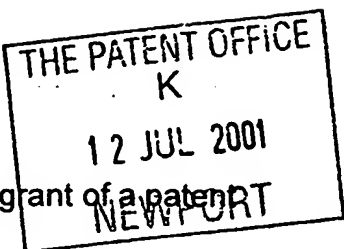
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# The Patent Office

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Request for grant of a patent

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1.	Your reference	GB920010034GB1		
2.	Patent application number (The Patent Office will fill in this part)	12 JUL 2001	0116987.9	
3.	Full name, address and postcode of the or of each applicant ( <i>underline all surnames</i> )	INTERNATIONAL BUSINESS MACHINES CORPORATION Armonk New York 10504 United States of America		
	Patents ADP number ( <i>if you know it</i> )	00519637001		
	If the applicant is a corporate body, give the country/state of its incorporation	State of New York United States of America		
4.	Title of the invention	A YIELD MANAGEMENT METHOD AND SYSTEM		
5.	Name of your agent ( <i>if you have one</i> )	R J Burt		
	"Address for Service" in the United Kingdom to which all correspondence should be sent ( <i>including the postcode</i> )	IBM United Kingdom Limited Intellectual Property Department Hursley Park Winchester Hampshire SO21 2JN		
	Patents ADP number ( <i>if you know it</i> )	07903925001		
6.	If you are declaring priority from one or more earlier patent applications, give the country and the date of filing of the or of each of these earlier applications and ( <i>if you know it</i> ) the or each application number	Country	Priority App No ( <i>if you know it</i> )	Date of filing ( <i>day/month/year</i> )
7.	If this application is divided or otherwise derived from an earlier UK application, give the number and the filing date or the earlier application	No of earlier application	Date of filing ( <i>day/month/year</i> )	

8. Is a statement of inventorship and of right to grant of a patent required in support of this request? (Answer 'Yes' if:  
a) any applicant named in part 3 is not an inventor, or  
b) there is an inventor who is not named as an applicant, or  
c) any named applicant is a corporate body.)
- Yes

9. Enter the number of sheets for any of the following items you are filing with this form. Do not count copies of the same document

Continuation sheets of this form

Description 30 ✓

Claim(s) 6 ✓

Abstract 1 ✓

Drawing(s) 8 + 8 e)

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Priority documents

Translations of priority documents

Statement of inventorship and right to grant of a patent (Patents Form 7/77) 3 ✓

Request for preliminary examination and search (Patents Form 9/77)

Request for substantive examination (Patents Form 10/77)

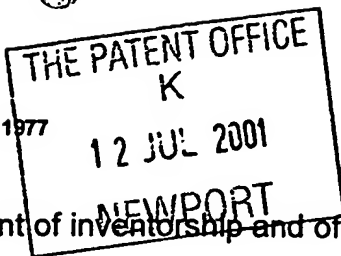
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11. I/We request the grant of a patent on the basis of this application

  
Signature

11 July 2001  
Date

12. Name and daytime telephone number of person to contact in the United Kingdom
- R J Burt  
01962 816646



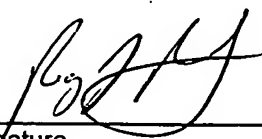
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Statement of inventorship and of right to grant of a patent

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- 
1. Your reference GB920010034GB1
- 
2. Patent application number  
(if you know it) 12 JUL 2001 0116987.9
- 
3. Full name of the or of each applicant INTERNATIONAL BUSINESS MACHINES CORPORATION
- 
4. Title of invention A YIELD MANAGEMENT METHOD AND SYSTEM
- 
5. State how the applicant(s) derived the right from the inventor(s) to be granted a patent By employment and by agreement
- 
6. How many, if any, additional Patents Forms 7/77 are attached to this form?
- 
7. I/We believe that the person(s) named over the page (and on any extra copies of this form) is/are the inventor(s) of the invention which the above patent application relates to.
-   
Signature
- 11 July 2001  
Date
- 
8. Name and daytime telephone number of person to contact in the United Kingdom R J Burt  
Tel: 01962 816646

Enter the full names, addresses and postcodes of the inventors in the boxes and underline the surnames

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If there are more than three inventors, please write their names and addresses on the back of another Patents Form 7/77 and attach it to this form

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## A YIELD MANAGEMENT METHOD AND SYSTEM

The present invention relates to a yield management method and system.

5       Yield Management Systems (YMSs) are commonly used as computer-based supports to facilitate decisions in several applications, such as in the transportation business and, in particular, in the airline industry. The goal of a yield management system is that of allocating an offered capacity to different types of requests (categories), in order to optimise  
10 a yield parameter. For example, the yield management system is used to maximise the revenue that can be obtained from selling a given transportable weight and volume of a cargo flight; the objective is to fly an aircraft as profitable as possible without allowing low-fare freights to displace high-fare freights.

15       All yield management systems use a forecast method to estimate the requests of the different categories; a decisional system is then applied to the estimated requests for determining a suggestion that should be given to maximise the yield parameter. Typically, stochastic or  
20 probabilistic methods are employed, in order to take into account an uncertainty of the estimated requests (and not only their mean value).

      A solution known in the art consists of estimating a simple probabilistic distribution of the variables involved in the yield  
25 management system. A stochastic linear programming model is then applied using the simple probabilistic distribution to determine an allocation model that maximizes a target function defining the yield parameter. An example of the aforementioned method is described in Williamson, E. L. "Airline Network Seat Inventory Control: Methodologies and Revenue  
30 Impact", Massachusetts Institute of Technology Phd Thesis - Cambridge (Mass) June 1992.

      A drawback of the solutions known in the art is that they become computationally intractable when a complexity of the problem defined by  
35 the yield management system increases. This is particular critical if the offered capacity is defined by more than one variable, such as the weight and the volume of a cargo flight. In these cases many simplifications, shortcuts and assumptions are introduced, in order to solve the problem in a reasonable amount of time. For example, the number of categories is  
40 limited or a manageable form of the probabilistic distributions is assumed (for example a Gauss, Poisson or uniform distribution). Moreover, the

known solutions discard some capacity variables (for example using only the weight and just checking a posteriori if volume constraints are satisfied) or discard the stochastic nature of the yield parameter (for example considering only its mean value).

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However, all the simplifications described above strongly reduce the accuracy of the yield management system. As a consequence, the solutions known in the art may lead to a relevant economic lost.

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It is an object of the present invention to overcome the above-mentioned drawbacks. In order to achieve this object, a method as set out in the first claim is proposed.

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Briefly, the present invention provides a yield management method for optimising a yield parameter resulting from assigning a capacity offered by a future instance of a service to each one of a plurality of different categories of requests competing for the capacity, the capacity being defined by at least one capacity variable, the method including the steps of: storing a set of historical profiles for each one of a plurality of previous instances of the service, the set including a historical profile of an historical value of each capacity variable reserved by each category, assigning a probability to each previous instance of the service, estimating a potential profile of a potential value of the capacity variable from each historical profile according to a corresponding current value of the capacity variable reserved by the category for the future instance of the service and according to a corresponding unconstrained demand of the capacity variable for the category in the previous instance of the service, defining a historical scenario for each previous instance of the service, the historical scenario including a final potential capacity variable from each corresponding potential profile, and determining an authorisation to allocate the offered capacity for each capacity variable of each category in the future instance of the service by applying a stochastic model to the historical scenarios according to the corresponding probabilities.

Moreover, the present invention also provides a computer program application for performing the method, a program product storing the program application, and a corresponding yield management system.

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Further features and the advantages of the solution according to the present invention will be made clear by the following description of a

preferred embodiment thereof, given purely by way of a non-restrictive indication, with reference to the attached figures, in which:

Fig.1 is basic block diagram of a data processing system in which the  
5 yield management method of the invention can be used;

Fig.2 shows a partial content of a working memory included in the  
system;

10 Figg.3a-3d are a flow chart describing the logic of the yield  
management method implemented in the system;

Figg.4a-4b depict different diagrams of some variables involved in  
the method.

15 With reference in particular to Fig.1, there is shown a data  
processing system 100, which is used as a computerised support for  
controlling cargo flights of an airline. The system 100 is formed by a  
reservation subsystem 105 and an authorisation subsystem 110; the  
20 subsystems 105 and 110 are connected to each other through a communication  
channel 115.

The reservation subsystem 105 consists of a mainframe having a  
Central Processing Unit (CPU) 120. A working memory (RAM) 125 and a bulk  
25 memory 130 (generally consisting of one or more hard-disks) are associated  
with the CPU 120. Moreover, a telecommunication control unit (TCU) 135 is  
used to connect multiple remote terminals 140 and the communication  
channel 115 to the CPU 120. The reservation subsystem 105 hosts a  
transactional processing environment, which allows the remote terminals  
30 140 to respond immediately to each request entered by an operator.

The authorisation subsystem 110 consists of a workstation having a  
RISC architecture. The authorisation subsystem 110 includes a Central  
Processing Unit (CPU) 150 and a working memory (RAM) 155 that is used  
35 directly by the CPU 150. The CPU 150 is associated with a bulk memory  
consisting of a hard-disk 160 and of a driver unit 165 for reading CD-ROMs  
170. Moreover, the CPU 150 is connected to an input unit (IN) 175, which  
consists for example of a keyboard and a mouse, and to an output unit  
(OUT) 180, which consists for example of a monitor and a printer. A  
40 network interface card (NIC) 185 is used to connect the communication  
channel 115 to the CPU 150.



The above-described architecture implements a yield management system. Yield management systems are a well-known application field of several Operation Research (OR) techniques. Basically, the goal of a yield management system is that of optimising one or more parameters that can be obtained from a given capacity of a generic service. A demand consisting of multiple types of requests (categories or classes) competes for the offered capacity; each category is defined by a subset of the requests exhibiting similar characteristics (such as a physical property, a fare, a reservation/cancellation behaviour).

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The yield management system selects the requests by accepting only the more convenient ones; this is done by assigning a booking limit (authorisation) to each category. The offered capacity may be allocated to the different categories using several techniques. In an out of nesting policy, the offered capacity is simply split among the categories according to their authorisations. This is a very conservative technique, which ensures a pre-set capacity to each category; however, it may lead to a yield loss when the requests for a category are lower than the respective authorisation. Conversely, in a nesting policy the same capacity may be reserved by more than one category; the categories are ranked according to their value, in order to assign a greater capacity to the most valuable ones. Particularly, in a total nesting policy each category may reserve all the capacity assigned to the category itself and to the categories with a lower value. This is a very aggressive technique, which favours the most valuable categories to the detriment of the other ones; however, it provides good results only when the requests for low-fare freights are received before the ones for high-fare freights. Otherwise, in a partial nesting policy each category may reserve the capacity assigned to the categories with a lower value only when the capacity assigned to the category itself has been completely allocated.

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In the example at issue, the yield management system is used to maximise a revenue  $\Lambda$  that can be obtained from a given capacity that is offered by each cargo flight; the offered capacity is defined by two different variables consisting of a total weight  $P_{tot}$  and of a total volume  $V_{tot}$  of the freights that may be transported by the flight. The freights are classified in five categories  $C_i$  (with  $i=1..5$ ), for example defined as normal light, special light, normal heavy, special heavy, and extra; this number is a good compromise between the opposed requirements of having uniform characteristics within each category (leading to a high number of categories) and a constant behaviour of each category (leading to a low

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number of categories).

The yield management system employs a partial nesting policy, with two sets of independent authorisations. A set of nesting authorisations  $Ap_i$  is used for the weight, with the categories that are ranked in a respective descending nesting order  $ORDp$  (from 1 to 5); a set of nesting authorisations  $Av_i$  is used for the volume, with the categories that are ranked in another independent nesting order  $ORDv$ . The partial nesting policy is preferred in this application since the period on which each flight is open for reservation is short, so that the order of the requests is very random. Moreover, it is generally difficult to rank the freights according to their value in an accurate manner, because both the weight and the volume are to be taken into account. In addition, the weight and the volume are not always constraints limiting the offered capacity at the same time.

As described in the following, the weight nesting authorisations  $Ap_i$ , the volume nesting authorisations  $Av_i$ , the weight nesting order  $ORDp$  and the volume nesting order  $ORDv$  for a future instance of each flight are determined by the authorisation subsystem 110. The authorisation subsystem 110 further supplies a minimum selling price, which consists of a couple of values indicating a weight unitary revenue  $M\lambda p$  and a volume unitary revenue  $M\lambda v$  for each segment of a network supported by the airline, under which a request should be refused because not profitable. The nesting authorisations  $Ap_i, Av_i$ , the nesting orders  $ORDp, ORDv$  and the minimum selling price  $M\lambda p, M\lambda v$  are transmitted to the reservation subsystem 105 and stored on its hard disk 130. The hard disk 130 further stores booking information  $BK$  about every request accepted by the reservation subsystem 105 for the future instance of each flight, such as a date of the accepted request, a weight and a volume of the freights, a name of the client, an applied fare (if already available). The sum of the weight and volume of the accepted requests gives a weight  $Rp_i$  and a volume  $Rv_i$ , respectively, already reserved by each  $i$ -th category. The reservation subsystem 105 calculates a residual weight  $Dp_i$  and a residual volume  $Dv_i$  that are still available for the  $i$ -th category according to the following formulas:

$$Rp_i = \min_{j \leq i} \left\{ \sum_{k \geq j} (Ap_k - Rp_k) \right\}$$

$$Rv_i = \min_{j \leq i} \left\{ \sum_{k \geq j} (Av_k - Rv_k) \right\}$$

The operator at a remote terminal 140 enters a reservation request for a particular flight. The reservation request is accepted if both the weight and the volume of the freights do not exceed the residual weight  $Dp_i$  and the residual volume  $Dv_i$ , respectively, of the corresponding category; otherwise, the reservation request is denied. At the same time, the reservation subsystem 105 adds together the minimum selling price of all the segments from an origin to a destination of the request, and multiplies the result by the weight and the volume of the freights; when the revenue of the request (if available) is lower than the value so obtained, the request is always refused. When the reservation request is accepted, it is recorded on the hard disk 130 and the respective residual capacities  $Dp_i, Dv_i$  are accordingly updated. The booking information BK, the reserved capacities  $Rp_i, Rv_i$  and the residual capacities  $Dp_i, Dv_i$  for the future instance of each flight are sent to the authorisation subsystem 110 either periodically (for example at the end of each day) or on request.

Similar considerations apply if the data processing system has a different architecture, for example consisting of a single computer or with the reservation subsystem and the authorisation subsystem working in the INTERNET, if the reservation subsystem and the authorisation subsystem have a different structure, for example if they consist of Personal Computers (PCs) or they are implemented with a local area network, if the freights are classified in a different number of categories, if other categories are envisaged, if a different nesting policy is employed (for example a mixed partial one), and so on.

Considering now Fig.2, there is shown a partial content of the working memory 155 of the authorisation subsystem; the information (programs and data) is typically stored on the hard-disk and loaded (at least partially) into the working memory 155 when the programs are running, together with an operating system and other application programs (not shown in the figure). The programs are initially installed onto the hard disk of the authorisation subsystem from CD-ROM.

An input/output interface (I/O) 205 controls interaction of a space controller with the authorisation subsystem. A network interface (NET) 210 processes a set of protocol layers, which allows the authorisation subsystem to exchange information with the reservation subsystem (through the communication channel).

The working memory 155 stores a historical repository 215 for each

flight. The historical repository 215 logs information about a series of previous instances of the flight; preferably, the last 104 previous instances of the flight (one per week in two years) are logged.

Particularly, the historical repository 215 includes a field  $INFO_j$  (with  $j=1..104$ ) that identifies the previous instance of the flight (for example its number and departure time), and a field  $SPH_j$  with information about each shipment (for example the weight and the volume of the freights that have been transported, and a fare actually applied). The historical repository 215 further stores a set of historical profiles, which consist of a temporal profile of the weight and of the volume that have been reserved by each category during an opening period of the previous instance of the flight. Each historical weight profile and each historical volume profile is defined by a series of daily snapshots (for example taken at the end of the day); if the previous instance of the flight was open 25 days before its departure, the  $k$ -th snapshot (with  $k=-25..0$ ) for the  $i$ -th category in the  $j$ -th historical weight profile consists of a reserved weight  $Rp_{ijk}$  and a residual weight  $Dp_{ijk}$ , whereas the same snapshot in the  $j$ -th historical volume profile consists of a reserved volume  $Rv_{ijk}$  and a residual volume  $Dv_{ijk}$ ; the first snapshot gives the reserved capacity  $Rp_{ij-25}$ ,  $Rv_{ij-25}$  and the residual capacity  $Dp_{ij-25}$ ,  $Dv_{ij-25}$  at the opening of the previous instance of the flight, whereas a last snapshot gives the reserved capacity  $Rp_{ij0}$ ,  $Rv_{ij0}$  and the residual capacity  $Dp_{ij0}$ ,  $Dv_{ij0}$  at its departure.

The working memory 155 further stores a current repository 220 for the future instance of each flight. Particularly, the current repository 220 includes a field  $INFO$  that describes the future instance of the flight (for example the offered capacity  $P_{tot}$ ,  $V_{tot}$  and its scheduled departure time), a field with the corresponding booking information  $BK$ , and a field with a current time  $t$  at which the information is taken. The current repository 220 further stores a set of current profiles, which consist of a temporal profile of the weight and of the volume that have been currently reserved by each category for the future instance of the flight. Each current weight profile and each current volume profile is defined by a series of daily snapshots. The  $k$ -snapshot (with  $k$  ranging from  $-25$  to the current time  $t$ ) for the  $i$ -th category in the current weight profile consists of a reserved weight  $Rp_{ik}$  and a residual weight  $Dp_{ik}$ , whereas the same snapshot in the current volume profile consists of a reserved volume  $Rv_{ik}$  and a residual volume  $Dv_{ik}$ ; the last snapshot gives the reserved capacity  $Rp_{it}$ ,  $Rv_{it}$  and the residual capacity  $Dp_{it}$ ,  $Dv_{it}$  at the current time  $t$ .

The booking information BK, the current reserved capacities  $Rp_{it}, Rv_{it}$  and the current residual capacities  $Dp_{it}, Dv_{it}$  are received from the reservation subsystem through the network interface 210. The set of profiles stored in the current repository 220 for each flight are copied  
 5 to the corresponding set of profiles in the historical repository 215 after departure of the flight; the shipment field SHP<sub>j</sub> is continually updated by the input/output interface 205 as soon as the corresponding information becomes available.

10 The historical repository 215 and the current repository 220 are accessed by a weighting module 225, which assigns a weight  $w_j$  to each previous instance of the flight. The weight  $w_j$  depends on a relevance of the previous instance of the flight with respect to its future instance.

15 The weights  $w_j$  are input to an estimation module 230, which further accesses the historical repository 215 and the current repository 220. The estimation module 230 determines a potential (weight and volume) profile from each corresponding historical profile. The potential profile is obtained adjusting the historical profile according to the respective  
 20 current profile and an estimated unconstrained (or untruncated) demand of the respective capacity variable for the category in the old instance of the flight. The unconstrained demand takes into account the periods of time in which the category has been closed to sales (because the reserved weight and/or the reserved volume for the category have reached their  
 25 booking limits defined by the respective nesting authorisations), and estimates a demand not limited by the offered capacity. The estimation of the unconstrained demand is carried out independently for the weight and for the volume. Each potential profile consists of 25 snapshots; the k-th snapshot for the i-th category in the j-th potential weight profile  
 30 consists of a potential weight request  $\overline{Rp}_{ijk}$ , whereas the same snapshot in the j-th potential volume profile consists of a potential volume request  $\overline{Rv}_{ijk}$ . The last snapshot of the potential weight profile and of the potential volume profile define a final potential weight request  $\overline{Rp}_{ij0}$  and a final potential volume request  $\overline{Rv}_{ij0}$ , respectively, for each i-th  
 35 category in the j-th previous instance of the flight.

The estimation module 230 further determines a potential revenue  $\overline{\Lambda}_{ij}$  for each i-th category transported by the j-th previous instance of the

flight. The potential revenue is estimated according to the revenue of the category in the previous instance of the flight; this value is further adjusted to take into account the revenue of the corresponding capacity currently reserved for the future instance of the flight.

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The estimation module 230 outputs a series of historical scenarios for each flight. The  $j$ -th scenario consists of the corresponding final potential weight request  $\overline{Rp}_{j0}$ , final potential volume request  $\overline{Rv}_{j0}$ , and potential revenue  $\overline{\Lambda}_j$  for each  $i$ -th category. The space controller defines a user scenario through the input/output interface 205. The user scenario consists of a potential weight request  $\overline{Rp}_i$ , a potential volume request  $\overline{Rv}_i$ , and a potential revenue  $\overline{\Lambda}_i$ , which are independently forecast by the space controller for each  $i$ -th category. A weight  $w$  is assigned to the user scenario by the space controller through the input/output interface 205.

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The weight  $w_j$  of each historical scenario  $\{\overline{Rp}_{j0}, \overline{Rv}_{j0}, \overline{\Lambda}_j\}$  and the weight  $w$  of the user scenario  $\{\overline{Rp}_i, \overline{Rv}_i, \overline{\Lambda}_i\}$ , which will be denoted as a whole with  $w_s$  in the following, are input to a normalisation module 235.

20 The normalising module 235 determines a probability  $\overline{w}_s$  of each (historical and user) scenario.

The probabilities  $\overline{w}_s$  are input to a nesting order module 240, which also receives the potential revenues  $\overline{\Lambda}_j$  of each historical scenario from the estimation module 230 and the potential revenues  $\overline{\Lambda}_i$  of the user scenario from the input/output interface 205. An input weight nesting order  $IN\_ORDp$  and an input volume nesting order  $IN\_ORDv$  are provided by the space controller thorough the input/output interface 205, and then transmitted to the nesting order module 240. The nesting order module 240 outputs the nesting orders  $ORDp, ORDv$  for the future instance of the flight.

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The nesting orders  $ORDp, ORDv$  are provided to an optimisation module 245; the optimisation module 245 further receives the historical scenarios

$\{\overline{Rp}_{y0}, \overline{Rv}_{y0}, \overline{\Lambda}_y\}$  from the estimation module 230 and the user scenario  
 $\{\overline{Rp}_i, \overline{Rv}_i, \overline{\Lambda}_i\}$  from the input/output interface 205, together with the  
respective probabilities  $\overline{w}_i$  from the normalisation module 235. The space  
controller supplies an aggressiveness parameter AG, indicative of an  
attitude to the risk, to the optimisation module 245 through the  
input/output interface 205. The optimisation module 245 employs stochastic  
programming techniques to determine the (optimal) nesting authorisations  
 $Ap_i, Av_i$  that maximise a potential revenue  $\overline{\Lambda}$  of the future instance of the  
flight.

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Stochastic programming techniques are commonly used as a support in  
decision-making processes under uncertainty, such as in financial trading.  
An example of stochastic programming technique is described in "IBM  
Stochastic Solution", Tito A. Ciriani, Stefano Gliozzi, AIROnews Bulletin  
of AIRO, VI, n.1 - Spring 2000, pp. 6-10. In particular, the module 245  
optimises each scenario using a deterministic linear programming  
technique; the results are then combined weighting each scenario according  
to the respective probability.

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Similar considerations apply if a whole application program (defined  
by the different modules described above) and the corresponding data are  
structured in a different manner, if other functions are provided, if the  
historical and current repository are replaced by equivalent memory  
structures, and so on.

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With reference now to Figg.3a-3d, the application program when  
running on the authorisation subsystem performs a method 300. The method  
starts at block 302 and then passes to block 304, wherein a menu with a  
series of possible choices is displayed on the monitor of the  
authorisation subsystem.

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The method carries out the operations corresponding to the selected  
choice. Particularly, if the space controller has selected a management  
function, the blocks 305-307 are executed, whereas if the space controller  
has selected an optimisation function, the blocks 308-374 are executed;  
conversely, if the space controller has selected an exit option the method  
ends at the final block 376.

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Considering now block 305 (management function), the information

about the future instance of the flight is received from the reservation subsystem, and it is stored into the current repository. The method passes to block 306, wherein the set of profiles stored in the current repository are copied to the historical repository after departure of the flight.

- 5 Continuing to block 307, the information about a selected shipment for the previous instance of the flight is updated as soon as it is available. The method then returns to block 304 waiting for a new command.

- 10 With reference to block 308 (optimisation function), a flight is selected either automatically (in a batch processing of all the flights of the airline) or by the space controller (in an interactive processing).

- 15 The method then assigns the corresponding weight  $w_j$  to the previous instances of the flight. Particularly, a season coefficient  $ws_j$  for each  $j$ -th previous instance of the flight is calculated at block 308. The coefficient  $ws_j$  depends on a season of the previous instance of the flight, that is, on a difference between a period of the previous instance of the flight and its future instance; the more the previous instance of the flight has happened in a similar period of the year as its future
- 20 instance, the more the season coefficient  $ws_j$  is bigger. The season coefficient  $ws_j$  is obtained from a matrix having 52 rows and 52 columns, which is preferably symmetric; each element  $[u,v]$  of the matrix (with  $u,v=1..52$ ) stores the season coefficient  $ws_j$  of a previous instance of the flight that departed in the  $u$ -th week of the year, for a future instance
- 25 of the flight that will depart in the  $v$ -th week of the year; the season coefficients  $ws_j$  have a value ranging from 0 to 1, with all the diagonal elements of the matrix ( $u=v$ ) that are equal to 1.

- Descending into block 312, a time distance coefficient  $wg_j$  for each
- 30  $j$ -th previous instance of the flight is calculated. The coefficient  $wg_j$  depends on a space between the current time and the departure of the previous instance of the flight; the more recent the previous instance of the flight is, the bigger the time distance coefficient  $wg_j$  is. The time distance coefficient  $wg_j$  is defined as a hyperbolic function of a number of
- 35 days  $g$  that have passed from the departure of the previous instance of the flight. We set  $wg_j=1$  for  $g=0$  and  $wg_j=a_1$  for  $g \rightarrow \infty$  (with  $0 \leq a_1 < 1$ , and preferably  $a_1=0$ ); moreover, we set an exponent  $a_2$  of the exponential law (with  $a_2 < 1$ , and preferably  $a_2=0.2$ ), and a scale coefficient  $a_3$  for reducing a concavity of the hyperbola (with  $a_3 \geq 30$ , and preferably  $a_3=30$ ). The time
- 40 distance coefficient  $wg_j$  is then given by:



$$wg_j = a_1 + \frac{1}{\left[ \frac{g}{a_3} + \left( \frac{1}{1-a_1} \right)^{\frac{1}{a_2}} \right]^{a_2}}$$

5 This results in a time distance coefficient  $wg_j \approx 0.53$  for  $g=708$  (2 years), so as to give more relevance to the most recent previous instances of the flight, while maintaining some relevance for the oldest previous instances of the flight at the same time.

10 The weight  $w_j$  for each  $j$ -th previous instance of the flight is then calculated at block 314, by combining the corresponding season coefficient  $ws_j$  and time distance coefficient  $wg_j$ . Particularly, the value  $w_j$  is calculated as a sum of the coefficients  $ws_j$  and  $wg_j$ , which are weighted according to a parameter  $h$  with a value between 0 and 1, and preferably  
15 equal to 0.65:

$$w_j = h \cdot ws_j + (1-h) \cdot wg_j$$

20 Considering now block 316, an opening coefficient  $a_{ijk}$  for each  $k$ -th snapshot of the  $i$ -th category in the  $j$ -th previous instance of the flight is estimated; the opening coefficient  $a_{ijk}$  indicates a percentage of time during which the category has been open to sales during a period associated with the snapshot, that is from a preceding snapshot ( $k-1$ ) to a current snapshot ( $k$ ).

25 As shown in Fig.4a, the category is open only when the residual weight and the residual volume are both positive. We denoted with  $s^*(f)$  a function that determines the sign of its argument  $f$ , that is:

30 
$$s^*(f) = \begin{cases} 0 & \text{if } f \leq 0 \\ 1 & \text{if } f > 0 \end{cases}$$

The starting conditions that are used (in the given order) to determine whether the category has been always open or always closed are:

35 
$$a_{ijk} = \begin{cases} 1 & \text{if } s^*(Dp_{ijk-1}) = s^*(Dp_{ijk}) = s^*(Dv_{ijk-1}) = s^*(Dv_{ijk}) = 1 \\ 0 & \text{if } s^*(Dp_{ijk-1}) = s^*(Dp_{ijk}) = 0 \text{ or } s^*(Dv_{ijk-1}) = s^*(Dv_{ijk}) = 0 \end{cases}$$

Otherwise, we calculate the sign of the derivative of the functions defined by the residual weight and the residual volume:

$$sp'_{ijk} = s^+(Dp_{ijk}) - s^+(Dp_{ijk-1})$$

$$sv'_{ijk} = s^+(Dv_{ijk}) - s^+(Dv_{ijk-1})$$

5

When  $sp'_{ijk}, sv'_{ijk} \neq 0$ , the intersection of these functions with a time axis is given by:

$$tp_{ijk} = \frac{Dp_{ijk-1}}{Dp_{ijk-1} - Dp_{ijk}}$$

$$tv_{ijk} = \frac{Dv_{ijk-1}}{Dv_{ijk-1} - Dv_{ijk}}$$

10

We now define the following functions:

15

$$\overline{tp}_{ijk} = \begin{cases} tp_{ijk} & \text{if } sp'_{ijk} = -1 \\ 1 - tp_{ijk} & \text{if } sp'_{ijk} = +1 \end{cases}$$

$$\overline{tv}_{ijk} = \begin{cases} tv_{ijk} & \text{if } sv'_{ijk} = -1 \\ 1 - tv_{ijk} & \text{if } sv'_{ijk} = +1 \end{cases}$$

20

If we consider all the possible cases, we can demonstrate that:

$$a_{ijk} = \begin{cases} \overline{tp}_{ijk} & \text{if } sp'_{ijk} \neq 0 \text{ and } sv'_{ijk} = 0 \\ \overline{tv}_{ijk} & \text{if } sv'_{ijk} \neq 0 \text{ and } sp'_{ijk} = 0 \\ \min\{\overline{tp}_{ijk}, \overline{tv}_{ijk}\} & \text{if } sp'_{ijk} \cdot sv'_{ijk} > 0 \\ \max\{0, (\overline{tp}_{ijk} + \overline{tv}_{ijk}) - 1\} & \text{otherwise} \end{cases}$$

25

The method continues to block 318, wherein a weight emphasis value  $Ep_{ik}$  and a volume emphasis value  $Ev_{ik}$  to be applied to the  $k$ -th snapshot for each  $i$ -th category (irrespective of the old instance of the flight) are calculated; the emphasis values  $Ep_{ik}, Ev_{ik}$  indicate what could have happened if the category had been always open to sales in the period associated with the  $k$ -th snapshot. The emphasis is calculated as a function of observations consisting of the demand of the respective category in the

30

corresponding historical profiles where the category was open in the same period.

- The method takes into account all the corresponding (weight or volume) historical profiles for the same category, which were always open in the period ( $a_{ijk}=1$ ). Further historical profiles (with a decreasing value of the opening coefficient  $a_{ijk}$ ) are also taken into account, if necessary, in order to ensure a sufficient number of observations (for example at least equal to 30% of the number of the previous instances of the flight).
- If we denote with  $P'_{ijk} = P_{ijk} - P_{ijk-1}$  and with  $V'_{ijk} = V_{ijk} - V_{ijk-1}$  a gradient of the reserved weight and of the reserved volume, respectively, the emphasis values  $Ep_{ik}, Ev_{ik}$  are defined as a weighted mean of these values:

$$Ep_{ik} = \frac{\sum_j w_{ijk} \cdot P'_{ijk}}{\sum_j w_{ijk}}$$

$$Ev_{ik} = \frac{\sum_j w_{ijk} \cdot V'_{ijk}}{\sum_j w_{ijk}}$$

- Passing to block 320, the method estimates a potential weight gradient  $Up_{ijk}$  and a potential volume gradient  $Uv_{ijk}$  for each  $k$ -th snapshot of the  $i$ -th category in the  $j$ -th historical weight profile and historical volume profile, respectively. The potential gradients  $Up_{ijk}, Uv_{ijk}$  are obtained as a linear interpolation between the value  $P'_{ijk}, V'_{ijk}$  for  $a_{ijk}=1$  and the value  $\max[P'_{ijk}, Ep_{ijk}] \max[V'_{ijk}, Ev_{ijk}]$  for  $a_{ijk}=0$ , that is:

$$Up_{ijk} = \max[P'_{ijk}, Ep_{ijk} + a_{ijk} + (P'_{ijk} - Ep_{ijk})]$$

$$Uv_{ijk} = \max[V'_{ijk}, Ev_{ijk} + a_{ijk} + (V'_{ijk} - Ev_{ijk})]$$

- The method then builds each potential profile. As shown in Fig.4b, the potential profile is set to the respective current profile up to a time  $t'$  corresponding to the current time  $t$ . Considering now block 322, the next snapshot for the  $i$ -th category in the  $j$ -th potential weight

profile is calculated integrating the potential weight gradient  $Up_{ijk}$ , whereas the same snapshot for the  $i$ -th category in the  $j$ -th volume profile is calculated integrating the potential volume gradient  $Uv_{ijk}$ ; the integration is carried out with a simple sum, that is  $\overline{Rp}_{ijk} = \overline{Rp}_{ijk-1} + Up_{ijk}$  and  $\overline{Rv}_{ijk} = \overline{Rv}_{ijk-1} + Uv_{ijk}$ , respectively. The potential gradients relating to the period including the time  $t'$  are reduced to take into account the time that has passed from the preceding snapshot; we denote with  $\bar{k}$  an index of the snapshot immediately following the time  $t'$ ; if  $g_t$  and  $g_{\bar{k}}$  are the number of hours from the time  $t'$  and from the  $\bar{k}$ -th snapshot to the departure of the previous instance of the flight, respectively, the potential gradients  $Up_{ijk}$ ,  $Uv_{ijk}$  are reduced according to the following formulas:

$$Up_{ijk} \leftarrow Up_{ijk} \cdot \frac{g_{\bar{k}} - g_t}{g_{\bar{k}} - g_{\bar{k}-1}}$$

$$Uv_{ijk} \leftarrow Uv_{ijk} \cdot \frac{g_{\bar{k}} - g_t}{g_{\bar{k}} - g_{\bar{k}-1}}$$

The method then checks at block 324 whether either the resulting potential weight request  $\overline{P}_{ijk}$  or the resulting potential volume request  $\overline{V}_{ijk}$  for the current snapshot is not strictly positive ( $\geq 0$ ). If so, the method descends into block 326, wherein the potential weight request  $\overline{P}_{ijk}$  and the potential volume request  $\overline{V}_{ijk}$  are both set to zero; the method then continues to block 328. Conversely, if both the resulting potential weight request  $\overline{P}_{ijk}$  and the resulting potential volume request  $\overline{V}_{ijk}$  are strictly positive, the method passes to block 328 directly. In other words, the potential weight capacity  $\overline{P}_{ijk}$  and the potential volume capacity  $\overline{V}_{ijk}$  (with  $k=t, \bar{k}, \dots, 0$ ) for the current snapshot are given by:

$$\overline{P}_{ijk} = \begin{cases} \overline{P}_{ijk-1} + Up_{ijk} & \text{if } \overline{P}_{ijk-1} + Up_{ijk} > 0 \text{ and } \overline{V}_{ijk-1} + Uv_{ijk} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\overline{V}_{ijk} = \begin{cases} \overline{V}_{ijk-1} + Uv_{ijk} & \text{if } \overline{P}_{ijk-1} + Up_{ijk} > 0 \text{ and } \overline{V}_{ijk-1} + Uv_{ijk} > 0 \\ 0 & \text{otherwise} \end{cases}$$

With reference now to block 328, the result is further corrected by reconciling the potential weight request  $\overline{P}_{ijk}$  and the potential volume request  $\overline{V}_{ijk}$  of the current snapshot, in order to resolve any inconsistency that may be introduced between the two capacity variables and that may bring about incongruent values of a density of the potential request. We denote with  $\delta_{ijk} = \frac{P_{ijk}}{V_{ijk}}$  the density of the reserved capacity for the i-th category in the k-th snapshot of the j-th previous instance of the flight, and with  $\bar{\delta}_{ijk} = \frac{\bar{P}_{ijk}}{\bar{V}_{ijk}}$  the density of the potential capacity for the same snapshot. If the density  $\bar{\delta}_{ijk}$  for the current snapshot is comprised between  $\delta_{ijk}$  and  $\bar{\delta}_{ijk-1}$ , not reconciliation is carried out and the method descends into block 330 (described in the following).

Otherwise, the potential demand is correct to an admissible reference density  $R\delta_{ijk}$  at block 332. The reference density  $R\delta_{ijk}$  is set to the value between  $\delta_{ijk}$  and  $\bar{\delta}_{ijk-1}$  that is closer to  $\bar{\delta}_{ijk}$ . Particularly, the potential weight request  $\overline{Rp}_{ijk}$  and the potential volume request  $\overline{Rp}_{ijk}$  are reconciled according to the following formula:

$$\bar{P}_{ijk} \leftarrow \frac{1}{2}(\bar{P}_{ijk} + \bar{V}_{ijk} \cdot R\delta_{ijk})$$

$$\bar{V}_{ijk} \leftarrow \frac{1}{2}(\bar{V}_{ijk} + \frac{\bar{P}_{ijk}}{R\delta_{ijk}})$$

Considering now block 330, the method checks whether the last snapshot (associated with the departure of the old instance of the flight) has been reached. If not, the method returns to block 324 for processing a next snapshot.

Conversely, the method descends into block 333, wherein a historical revenue  $\Lambda_{ij}$  for the i-th category transported by the j-th previous instance of the flight is calculated by summing a value of the respective shipments. The historical revenue  $\Lambda_{ij}$  may be calculated only when the value of all the shipments for the category are known; otherwise, if the value

of one or more shipments is not available, the historical revenue  $\Lambda_{ij}$  for the whole category is deemed unknown. In a similar manner, a current revenue  $\Lambda_i$  for the  $i$ -th category in the future instance of the flight is calculated by summing a value of the respective requests accepted at the current time  $t$ .

In the cargo flights, the freight fares are defined according to a so-called taxable weight  $P^*$ ; the taxable weight  $P^*$  is a virtual weight of the freights that is applied to calculate their value for a given unit weight fare. We denote with  $\Delta^*$  a reference density (for example  $\Delta^*=167$  kg/m<sup>3</sup>). If a density  $\delta$  of the freights is equal to or higher than  $\Delta^*$  (heavy freights) the taxable weight  $P^*$  of the freights is equal to their real weight; otherwise, if the density  $\delta$  of the freights is lower than  $\Delta^*$  (light freights), the taxable weight  $P^*$  of the freights is obtained multiplying their volume by the reference density  $\Delta^*$ :

$$P^* = \max(P, V \cdot \Delta^*)$$

A (weight) unit revenue  $\lambda$  is then calculated as  $\lambda = \frac{\Lambda}{P^*}$

We define a historical taxable weight for each  $i$ -th category in the  $j$ -th previous instance of the flight as  $Rp_{ij0}^* = \max(Rp_{ij0}, Rv_{ij0} \cdot \Delta^*)$ , and a current taxable weight for each  $i$ -th category as  $Rp_{iu}^* = \max(Rp_{iu}, Rv_{iu} \cdot \Delta^*)$ ; it is then possible to calculate a historical unit revenue  $\lambda_{ij}$  for each  $i$ -th category in the  $j$ -th previous instance of the flight and a current unit revenue  $\lambda_i$  for each  $i$ -th category in the future instance of the flight by applying the above mentioned formula, that is  $\lambda_{ij} = \frac{\Lambda_{ij}}{Rp_{ij0}^*}$  and  $\lambda_i = \frac{\Lambda_i}{Rp_{iu}^*}$ , respectively.

Proceeding to block 334, the historical unit revenues  $\lambda_{ij}$ , for each  $i$ -th category in the  $j$ -th previous instance of the flight wherein the respective historical revenue  $\Lambda_{ij}$  is unknown, are set to the weighted mean of the respective values  $\lambda_{ij}$  available, that is:

$$\lambda_{ij} = \frac{\sum_j w_j \lambda_{ij}}{\sum_j w_j}$$

The method then estimates a potential unit revenue  $\bar{\lambda}_{ij}$  for each i-th category in the j-th previous instance of the flight, by adjusting the corresponding historical unit revenue  $\lambda_{ij}$  according to the corresponding current unit revenue  $\lambda_i$ . Particularly, a departure coefficient wd for the future instance of the flight is calculated at block 336. The coefficient wd depends on a space between the current time t and the departure of the future instance of the flight. The more the current time is close to the departure, the bigger the coefficient wd is. We assume that the current unit revenue  $\lambda_i$  does not affect the historical unit revenue  $\lambda_{ij}$  when the number of days to the departure of the future instance of the flight is greater than a pre-set value  $g_m$  (for example  $g_m=10$ ). If we denote with  $g_d$  the number of days from the current time t to the departure of the future instance of the flight, the departure coefficient wd is given by:

$$wd = \min\left(1, \frac{g_d}{g_m}\right)$$

Passing to block 338, an increment coefficient  $wx_{ij}$  for each i-th category in the j-th previous instance of the flight is further calculated. The coefficient  $wx_{ij}$  depends on an increment of the potential capacity at the departure for the i-th category in the j-th previous instance of the flight with respect to the current reserved capacity. We define a potential taxable weight for each i-th category in the j-th previous instance of the flight as  $\bar{P}_{ij0} = \max(\bar{Rp}_{ij0}, \bar{Rp}_{ij0} \cdot \Delta^*)$ ; the increment coefficient  $wx_{ij}$  is then given by the following formula:

$$wx_{ij} = \begin{cases} \max\left(0, \frac{\bar{P}_{ij0} - P_u^*}{\bar{P}_{ij0}}\right) & \text{if } \bar{P}_{ij0} > 0 \\ 0 & \text{otherwise} \end{cases}$$

A corrective factor  $wc_{ij}$  for each i-th category in the j-th previous instance of the flight is then calculated at block 340, by combining the corresponding departure coefficient wd and increment coefficient  $wx_{ij}$ . Particularly, the corrective factor  $wc_{ij}$  is calculated as a sum of the coefficients wd and  $wx_{ij}$ , which are weighted according to a parameter hc having a value between 0.5 and 1, and preferably equal to 0.6:

$$wc_{ij} = hc \cdot wx_{ij} + (1 - hc) \cdot wd$$

The potential unit revenue  $\overline{\lambda}_{ij}$  for the i-th category in the j-th old instance of the flight is then calculated at block 342 as a sum of the corresponding historical unit revenue  $\lambda_{ij}$  and current unit revenue  $\lambda_i$ , which are weighted according to the corrective factor  $w_{cij}$ , that is:

5

$$\overline{\lambda}_{ij} = w_{cij} \cdot \lambda_{ij} + (1 - w_{cij}) \cdot \lambda_i$$

The potential revenue  $\overline{\Lambda}_{ij}$  for each i-th category in the j-th old instance of the flight is simply obtained as  $\overline{\Lambda}_{ij} = \overline{\lambda}_{ij} \cdot \overline{Rp}_{ij0}$ .

10

Proceeding to block 344, the historical scenarios  $\{\overline{Rp}_{ij0}, \overline{Rv}_{ij0}, \overline{\Lambda}_{ij}\}$  are determined for each old instance of the flight. The method then checks at block 348 whether the space controller wishes to input a user scenario. If not, the method descends into block 350 (described in the following). Conversely, the space controller inputs the user scenario  $\{\overline{Rp}_i, \overline{Rv}_i, \overline{\Lambda}_i\}$  at block 352 and inputs its weight  $w$  at block 354; the method then passes to block 350.

15

Considering now block 350, the probability  $\overline{w}_s$  of each (historical and user) scenario is calculated normalising the corresponding weight  $w_s$ , that is:

20

$$\overline{w}_s = \frac{w_s}{\sum_{i=1}^{104, user} w_i}$$

25

The method then proceeds to block 355, wherein the input nesting orders IN\_ORDp, IN\_ORDv are provided by the space controller. Descending into block 356, the method checks whether for each i-th category some of the j-th historical scenarios have the final potential weight request  $\overline{Rp}_{ij0}$ , the final potential volume request  $\overline{Rv}_{ij0}$  and the potential revenue  $\overline{\Lambda}_{ij}$  all strictly positive. If not, the method passes to block 358 (described in the following). Conversely, the method enters block 360,

30



wherein a potential weight revenue  $\overline{\lambda p_{is}} = \frac{\overline{\Lambda_{is}}}{R p_{is0}}$  and a potential volume

revenue  $\overline{\lambda v_{is}} = \frac{\overline{\Lambda_{is}}}{R v_{is0}}$  are calculated for each i-th category that meets the

above mentioned conditions (taking into consideration all the historical

scenarios and the user scenario). A corresponding mean weight revenue  $\hat{\lambda p_i}$

5 and a corresponding mean volume revenue  $\hat{\lambda v_i}$  are then calculated for these categories as a weighted mean of the potential weight revenues  $\overline{\lambda p_{is}}$  and the potential volume revenues  $\overline{\lambda v_{is}}$ , respectively:

$$\hat{\lambda p_i} = \frac{\sum_s \overline{w_s} \cdot \overline{\lambda p_{is}}}{\sum_s \overline{w_s}}$$

$$\hat{\lambda v_i} = \frac{\sum_s \overline{w_s} \cdot \overline{\lambda v_{is}}}{\sum_s \overline{w_s}}$$

15 The method then passes to block 358.

Considering now block 358, the position of each category in the nesting orders ORDp, ORDv is then determined. For each capacity variable

20 (weight and volume), the categories that do not have a respective mean revenue maintain their position in the input nesting orders

IN\_ORDp, IN\_ORDv. The other categories having a respective mean revenue  $\hat{\lambda p_i}, \hat{\lambda v_i}$  are ranked in a corresponding descending order within the

available positions. For example, let us suppose that the input weight

25 nesting order for the five categories is IN\_ORDp={C<sub>2</sub>, C<sub>4</sub>, C<sub>3</sub>, C<sub>5</sub>, C<sub>1</sub>}. The categories C<sub>2</sub>, C<sub>3</sub> and C<sub>5</sub> are ranked in the order {C<sub>3</sub>, C<sub>5</sub>, C<sub>2</sub>} according to

their respective mean weight revenues  $\hat{\lambda p_3}, \hat{\lambda p_5}, \hat{\lambda p_2}$ , conversely, the mean

weight revenue  $\hat{\lambda p_4}$  (for the category C<sub>4</sub>) and the mean weight revenue  $\hat{\lambda p_1}$

(for the category C<sub>1</sub>) are not available. In this case, the resulting weight

30 nesting order is ORDp={C<sub>3</sub>, C<sub>4</sub>, C<sub>5</sub>, C<sub>2</sub>, C<sub>1</sub>}.

The method then proceeds to block 362, wherein the aggressiveness parameter AG is input by the space controller. The method builds a stochastic model that maximises a target function defined by the potential revenue  $\bar{\Lambda}$  of the future instance of the flight; the potential revenue  $\bar{\Lambda}$  is calculated using both a nesting policy (irrespective of an order of the requests) and an out of nesting policy; the nesting policy (best case) and the out of nesting policy (worst case) are combined according to the aggressiveness parameter.

Particularly, a portion of the target function for the nesting policy is built at block 364. We define a potential nesting constrained weight  $\overline{TNp_{ij}}$  and a potential nesting constrained volume  $\overline{TNv_{ij}}$  that would have been transported for each i-th category by the j-th previous instance of the flight complying with the corresponding nesting authorisations  $Ap_i$  and  $Av_i$ , respectively:

$$\sum_{i \in I} \overline{TNp_{ij}} \leq Ap_i$$

$$\sum_{i \in I} \overline{TNv_{ij}} \leq Av_i$$

The variables  $\overline{TNp_{ij}}, \overline{TNv_{ij}}$  are subjected to the constraint

$$\overline{TNp_{ij}} = \frac{\overline{TNv_{ij}}}{\delta_{ij0}} \text{ and to the simple bounds } \overline{TNp_{ij}} \leq P_{off}, \overline{TNv_{ij}} \leq V_{off}.$$

A potential nesting revenue  $\overline{TN\Lambda_j}$  of the j-th previous instance of the flight that would have been produced by the potential nesting constrained weight  $\overline{TNp_{ij}}$  and the potential nesting constrained volume  $\overline{TNv_{ij}}$  is given by:

$$\overline{TN\Lambda_j} = \sum_{i=1}^5 \overline{TNp_{ij}} \cdot \hat{\lambda p_i}$$

The method then builds a further portion of the target function for the out of nesting policy, using two independent components for the weight and for the volume. Particularly, a set of out of nesting weight authorisations  $AOp_i$  and a set of out of nesting volume authorisations  $AOv_i$  are defined at block 366. If we denote with  $Pe$  the offered weight that is not used, and with  $Ve$  the offered volume that is not used, the out of

nesting authorisations  $AOp_i, AOv_i$  are defined by the following relations:

$$P = \sum_{i=1}^5 AOp_i + Pe$$

$$V = \sum_{i=1}^5 AOv_i + Ve$$

The out of nesting authorisations  $AOp_i$  and  $AOv_i$  are further constrained by the nesting authorisations  $Ap_i$  and  $Av_i$ , respectively:

$$\sum_{i=1}^5 Ap_i = \sum_{i=1}^5 AOp_i + Pe$$

$$\sum_{i=1}^5 Av_i = \sum_{i=1}^5 AOv_i + Ve$$

We now define a potential out of nesting constrained weight  $\overline{TOp_{ij}}$  that would have been transported for each  $i$ -th category by the  $j$ -th previous instance of the flight in the worst case complying with the out of nesting authorisations  $AOp_i, AOv_i$  only for the weight, that is:

$$\overline{TOp_{ij}} \leq AOp_i$$

$$\overline{TOp_{ij}} \leq AOv_i \cdot \overline{\lambda_{ij0}}$$

Even in this case, the potential out of nesting constrained weight  $\overline{TOp_{ij}}$  is subjected to the simple bounds  $\overline{TOp_{ij}} \leq P_{off}$ . A potential out of nesting weight revenue  $\overline{TOAp_j}$  of the  $j$ -th previous instance of the flight that would have been produced by the potential out of nesting constrained weight  $\overline{TOp_{ij}}$  is given by:

$$\overline{TOAp_j} = \sum_{i=1}^5 \overline{TOp_{ij}} \cdot \hat{\lambda}_{p_i}$$

Proceeding to block 368, we now define a potential out of nesting constrained volume  $\overline{TOv_{ij}}$  that would have been transported for each  $i$ -th category by the  $j$ -th previous instance of the flight in the worst case

complying with the out of nesting authorisations  $AOp_i, AOv_i$  only for the volume, that is:

$$\overline{TOv_{ij}} \leq AOv_i$$

$$\overline{TOv_{ij}} \leq \frac{AOp_i}{\lambda_{ij0}}$$

5

As above, the potential out of nesting constrained volume  $\overline{TOv_{ij}}$  is subjected to the simple bounds  $\overline{TOv_{ij}} \leq V_{tot}$ . A potential out of nesting volume revenue  $\overline{TO\Lambda v_j}$  of the j-th previous instance of the flight that would have been produced by the potential out of nesting constrained volume  $\overline{TOv_{ij}}$  is given by:

10

$$\overline{TO\Lambda v_j} = \sum_{i=1}^s \overline{TOv_{ij}} \cdot \hat{\lambda}_{p_i} \cdot \delta_{ij0}$$

15

The method then passes to block 370, wherein the target function to be optimised is defined combining the nesting portion and the weight and volume component of the out of nesting portion with the respective probabilities, according to the aggressiveness parameter AG:

20

$$\overline{\Lambda} = AG \cdot \sum_s (\overline{TNA_s} \cdot \overline{w_s}) + (1 - \frac{AG}{2}) \cdot \sum_s (\overline{TO\Lambda p_s} \cdot \overline{w_s}) + (1 - \frac{AG}{2}) \cdot \sum_s (\overline{TO\Lambda v_s} \cdot \overline{w_s})$$

25

The nesting authorisations  $Ap_i, Av_i$  that maximise the target function defined above (with the variables meeting the respecting constrains and single bounds) are then obtained at block 372.

30

Several by-products of this process may also be obtained. For example, the stochastic model calculates the minimum selling price  $M\Lambda p, M\Lambda v$ ; particularly, the value  $M\Lambda p$  is equal to a marginal weight revenue and the value  $M\Lambda v$  is equal to a marginal volume revenue in the neighbourhoods of the optimal solution, which are obtained as the partial derivative of the target function with respect to the offered weight and offered volume, respectively. Moreover, the stochastic model calculates a mean potential constrained weight  $\overline{CRp_{ij}}$  and a mean potential constrained volume  $\overline{CRv_{ij}}$  as:

$$\overline{CRp_{ij}} = AG \cdot \sum_s (\overline{TNp_s} \cdot \overline{w_s}) + (1 - AG) \cdot \sum_s (\overline{TOP_s} \cdot \overline{w_s})$$

$$\overline{CRv_{ij}} = AG \cdot \sum_s (\overline{TNv_s} \cdot \overline{w_s}) + (1 - AG) \cdot \sum_s (\overline{TOv_s} \cdot \overline{w_s})$$

5       The method verifies at block 374 whether a last flight to be optimised has been processed. If not, the method returns to block 308 for applying the operations described above to a next flight. Conversely, the method returns to block 304 waiting for a new command.

10       Similar considerations apply if the program application performs an equivalent method, for example with error routines, escape functions for stopping execution of the program, several concurrent processes that execute the above described operations in parallel, and the like. Alternatively, a resolution of the freights or a unit load device used for  
15       transporting the freights are considered (for example by reducing the residual capacity of a pre-set coefficient), no-show freights and/or denied boarding (off-loaded) freights are estimated, an overbooking is taken into account (for example increasing the offered capacity by a pre-set coefficient), or no by-product is calculated.

20       More generally, the present invention provides a yield management method, which is used to optimise a yield parameter resulting from assigning a capacity offered by a future instance of a service to each one of a plurality of different categories of requests competing for the  
25       capacity; the capacity is defined by one or more capacity variables. A set of historical profiles is stored for each one of a plurality of previous instances of the service; the set includes a historical profile of an historical value of each capacity variable reserved by each category. A probability is then assigned to each previous instance of the service. The  
30       method estimates a potential profile of a potential value of the capacity variable from each historical profile; the estimation is carried out according to a corresponding current value of the capacity variable reserved by the category for the future instance of the service and according to a corresponding unconstrained demand of the capacity variable  
35       for the category in the previous instance of the service. A historical scenario is defined for each previous instance of the service; the historical scenario includes a final potential capacity variable from each corresponding potential profile. The method then determines an authorisation to allocate the offered capacity for each capacity variable

of each category in the future instance of the service by applying a stochastic model to the historical scenarios according to the corresponding probabilities.

5       The computational complexity of the method according to the invention increases linearly with the number of the historical scenarios and it is independent of the number of categories; this is particularly advantageous in a commercial application wherein the market dynamic is very fast, so that a relatively low number of historical scenarios is often enough to  
10   take planning decisions. As a consequence, the problem defined by the yield management method can be solved in a reasonable amount of time. This allows the nesting authorisations to be recalculated several times during the opening period, and particularly with a higher frequency close to the departure of the future instance of the flight.

15       The solution of the invention does not require any simplification, shortcut or assumption to be made. Particularly, all the capacity variables are taken into account simultaneously, the scenarios are not artificially reduced in any way, and no assumption is made on the (simple  
20   and joint) statistical distribution of the variables involved in the problem.

      The envisaged solution derives the probabilistic nature of the demand directly from the reality. As a consequence, any kind of demand (even if  
25   particularly scattered or inconsistent) yields to consistent and conservative suggestions. Therefore, the method of the invention is accurate, robust and secure.

      The preferred embodiment of the invention described above offers  
30   further advantages. For example, the probability assigned to each historical scenario is calculated by normalising the corresponding weight, which is a combination of the season coefficient and the time distance coefficient. In this way, the previous instances of the flight that have happened in a similar period as their future instance and/or that are  
35   close to the current time are given more importance.

      Preferably, this value is calculated as a weighted sum of the season coefficient and the time distance coefficient. This approach gives more relevance to the dominant coefficient, and yields to a good result even if  
40   the number of previous instances of the flight is not very high.

Similar considerations apply if a different number of periods is taken into considerations (such as the days of the week), if the season coefficient and/or the time distance coefficient are calculated in a different manner, if different values are used in the formula defining the time distance coefficient and/or the weight, if the time distance coefficient depends on a release of the previous instance of the flight (or more generally on an occurrence thereof), and so on. Alternatively, each weight is assigned to a set consisting of more than one previous instance of the flight (such as including all the previous instances of the flight in a corresponding month), the weight further depends on mobile events (such as Easter), the weight is calculated as the product of the season coefficient and the time distance coefficient (giving a high weight only when both the coefficients are high).

The potential profiles are estimated by emphasising the demand when the respective category has been closed to sales; the emphasis values are defined in order to take into account both the weight and the volume in a symmetric manner. The suggested procedure emphasises the demand according to what has happened in the other previous instances of the flight in which the category was (almost) always open.

Moreover, the result of the integration is corrected to ensure that no negative forecasts for the potential requests at the departure of the future instance of the flight are obtained when the demand is low.

Similar considerations apply if each profile consists of a different number of snapshots, if the snapshots are taken with a different frequency, if the potential profiles are built with a different procedure, if the opening coefficients and/or the emphasis values are calculated in another manner, if a different interpolation technique for obtaining the potential gradients is employed, if the potential gradients are integrated in a different manner, if a dummy snapshot preceding the opening of the flight is introduced for processing flights that have not been opened yet, and the like.

Alternatively, the emphasis values are calculated taking into account only the previous instances of the flight in which the category was always open (assuming a sufficient number of corresponding historical profiles), using only the historical profiles with a growing demand, or performing a weighted mean of the more optimistic increments of the demand in the historical profiles. However, the method according to the present

invention leads itself to be carried out without any correction to the result of the integration, or even detecting and storing any denied request, so as to know the potential profiles directly (even if some adjustment is always suggested because it is very difficult to know all the denied requests for sure).

The potential profiles are further reconciled to an admissible reference density; in this way, it is assured that no inconsistent forecasts are produced.

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The reference density is set to the closest one between the density of the previous snapshot in the potential profile and the density of the current snapshot in the historical profile. This value makes it possible to take into account what has happened in the historical profile without excessively affecting the potential profile.

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Similar considerations apply if another logic relation between the capacity variables is considered for the reconciliation of the potential profiles, if the reconciliation is carried out in a different manner, if a different reference density is employed, and so on. For example, in an alternative embodiment of the present invention the reference density is set to a mean of the above-mentioned limit values, to the density of the preceding snapshot in the historical profile, or to a pre-set value. However, the method of the invention leads itself to be carried out even without performing any reconciliation of the potential profiles.

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Each historical scenario further includes the potential revenue, which is estimated from the corresponding historical revenue; when the value is unknown, the historical revenue is set to a weighted mean of the values known in the other previous instances of the flight. This is particularly advantageous when it is difficult, if not impossible, to detect the real revenue of each shipment in the previous instance of the flight at its departure; moreover, the employed procedure is independent of the hypothesis that the freights of each shipment maintain the same category from the historical profile to the potential profile.

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Preferably, the potential unit revenue is calculated as a mean of the historical unit revenue and the current unit revenue, weighted according to the corrective factor. In this way, the potential unit revenue takes into account what is currently happening for the future instance of the flight; however, the current unit revenue affects the potential unit

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revenue only when the corrective factor is high (this is preferable when the current revenue is not particularly accurate).

The corrective factor is a combination of the departure coefficient and the increment coefficient. In this way, the potential revenue takes into account what is actually happening only when this can affect the forecast.

The corrective factor is calculated as a weighted sum of the departure coefficient and the increment coefficient. This approach gives more relevance to the dominant coefficient, and yields to a good result even if the current revenue is not very accurate.

Similar considerations apply if the potential revenue is estimated in a different manner, if the value of each accepted request is set to a mean market value (when its value cannot be determined in an accurate manner by the reservation subsystem), if another formula is employed for combining the potential unit revenue and the current unit revenue, if the departure coefficient and/or the increment coefficient are calculated in a different manner, if different values are used in the formula defining the departure coefficient and/or the corrective factor, and so on. Alternatively, the corrective factor depends on other coefficients, the corrective factor is calculated as the product of the departure coefficient and the increment coefficient (giving a high value only when both the coefficients are high); however, the solution of the invention leads itself to be implemented without taking the current revenue into consideration, or even without estimating any potential revenue.

Two nesting orders (for the weight and the volume) are calculated according to the mean potential revenue of each category.

Moreover, corresponding input nesting orders are also provided; the nesting orders are obtained updating the input nesting orders according to the mean potential revenue for the categories in which this value is available.

Similar considerations apply if another technique is employed for calculating the nesting orders, or if the input nesting order are updated in a different manner. Alternatively, no input nesting orders are provided, or the nesting orders are not calculated by the system (but they are always set by the space controller).

The nesting portion and the out of nesting portion of the target function, which are combined using the aggressiveness parameter, allow the result of the optimisation to be tuned according to different attitudes to the risk; particularly, a more aggressive behaviour gives a higher revenue in the worst case (wherein the requests for the most valuable freights are the last ones), and a lower revenue in the best case (wherein the requests for the most valuable freights are the first ones).

Advantageously, the out of nesting portion of the target function is calculated with two independent components for the weight and for the volume. In this way, the constraints for the worst case are slightly relaxed with respect to a real out of nesting policy, wherein the out of nesting authorisations must be met by both the weight and the volume. Conversely, in the proposed formula the two capacity variables are considered one at the time, so that each request may be accepted even if it meets the out of nesting authorisations only for the weight and not for the volume (or vice-versa).

Similar considerations apply if the stochastic model is defined in a different manner, if another target function is envisaged, if the different components of the out of nesting portion and/or the two portions of the target function are combined in another way, and the like. However, the solution of the invention leads itself to be carried out even with the out of nesting portion that is calculated simultaneously for the weight and the volume, or with the target function that is defined with a single policy (either in nesting or out of nesting).

Preferably, the space controller inputs a user scenario with the respective probability; in this way, any personal feeling of the future may be introduced into the stochastic model in a very simple manner. However, the solution according to the present invention leads itself to be implemented even without the possibility of imputing any further scenario.

The proposed solution is particularly advantageous in a cargo flight, wherein the capacity is defined by two independent variables (weight and volume), and the revenue of the old instances of the flight is not immediately available. Alternatively, the solution of the invention is used to optimise another yield parameter resulting from assigning an offered capacity defined by one or more different variables; moreover, other applications of the yield management method are contemplated, for

example in a passenger flight, in a different transportation system (such as a train), in a system for controlling booking of rooms in a hotel, and the like.

5           Advantageously, the solution according to the present invention is implemented with a program application, which consists of a computer program (software) provided on CD-ROM.

10           Alternatively, the program application is provided on floppy-disk, is pre-loaded onto the hard-disk, or is stored on any other computer readable medium, is sent to the authorisation subsystem through the INTERNET, is broadcast, or more generally is provided in any other form directly loadable into the working memory of the authorisation subsystem. However, the method according to the present invention leads itself to be carried  
15 out even with a hardware structure, for example integrated in a chip of semiconductor material.

20           Naturally, in order to satisfy local and specific requirements, a person skilled in the art may apply to the solution described above many modifications and alterations all of which, however, are included within the scope of protection of the invention as defined by the following claims.

## CLAIMS

1. A yield management method (300) for optimising a yield parameter resulting from assigning a capacity offered by a future instance of a service to each one of a plurality of different categories of requests competing for the capacity, the capacity being defined by at least one capacity variable, the method including the steps of:

storing (306) a set of historical profiles for each one of a plurality of previous instances of the service, the set including a historical profile of an historical value of each capacity variable reserved by each category,

assigning (310-314,350) a probability to each previous instance of the service,

estimating (316-332) a potential profile of a potential value of the capacity variable from each historical profile according to a corresponding current value of the capacity variable reserved by the category for the future instance of the service and according to a corresponding unconstrained demand of the capacity variable for the category in the previous instance of the service,

defining (344) a historical scenario for each previous instance of the service, the historical scenario including a final potential capacity variable from each corresponding potential profile, and

determining (362-372) an authorisation to allocate the offered capacity for each capacity variable of each category in the future instance of the service by applying a stochastic model to the historical scenarios according to the corresponding probabilities.

2. The yield management method (300) according to claim 1, wherein the step of assigning the probability (310-314,350) includes:

determining (310) a first coefficient depending on a difference between a temporal period associated with the previous instance of the service and a temporal period associated with the future instance of the service,

determining (312) a second coefficient depending on a space between a current time and an occurrence time of the previous instance of the service,

calculating (314) a weight by combining the first coefficient and the second coefficient,

calculating (350) the probability by normalising the weight.

3. The yield management method (300) according to claim 2, wherein the

step of calculating the weight consists of calculating (314) a weighted sum of the first coefficient and the second coefficient.

4. The yield management method (300) according to any claim from 1 to 3, wherein each historical profile and each potential profile include a plurality of corresponding snapshots of the reserved capacity variable and of the potential capacity variable, respectively, the step of estimating the potential profile (316-332) including:

estimating (316) an opening coefficient for each period comprised between two consecutive snapshots, the opening coefficient being indicative of a percentage of time during which the category was open in the period,

calculating (318) an emphasis value for each period as a weighted mean of a gradient in the period of the reserved capacity variable for the category in a subset of the corresponding historical profiles,

estimating (320) a potential gradient for each period as a linear interpolation between the gradient, for a first value of the opening coefficient indicative of a complete opening of the category, and the highest between the gradient and the emphasis value, for a second value of the opening coefficient indicative of a complete closure of the category, and

constructing (322) the potential profile from a time corresponding to the current time by integrating the potential gradients starting from the corresponding current capacity variable.

5. The yield management method (300) according to claim 4, wherein the step of estimating the potential profile (316-332) further includes:

verifying (324) whether at least one result of the integration in each snapshot for each category is not strictly positive, and

setting (326) each potential capacity variable of the category in the snapshot to zero if the verification is affirmative.

6. The yield management method (300) according to claim 4 or 5, wherein the at least one capacity variable consists of a plurality of capacity variables, the step of estimating the potential profile (316-332) further including reconciling (328,332) the potential capacity variables for each category in the snapshot to a reference value of a logic relation therebetween.

7. The yield management method (300) according to claim 6, wherein the step of reconciling (328,332) includes:

verifying (328) whether the logic relation between the potential capacity variables for the category in the snapshot is included between a first limit defined by the logic relation between the reserved capacity variables for the category in the snapshot and a second limit defined by the logic relation between the potential capacity variables for the category in a preceding snapshot, and

updating (332) the potential capacity variables for the category in the snapshot for correcting the corresponding logic relation to the closest one of the first limit and the second limit.

8. The yield management method (300) according to any claim from 1 to claim 7, further including the steps of:

determining (333) a historical unit value of the yield parameter for each category in each previous instance of the service if available,

estimating (334) the historical unit yield parameter for each category in the other previous instances of the service as a weighted mean of the corresponding historical unit yield parameters available,

estimating (305,336-342) a potential unit value of the yield parameter for each category in each previous instance of the flight from the corresponding historical unit yield parameter, and

calculating (344) a potential value of the yield parameter for each category in each previous instance of the service multiplying the corresponding potential unit yield parameter by the corresponding potential capacity variable, the potential yield parameter being included in the corresponding historical scenario.

9. The yield management method (300) according to claim 8, wherein the step of estimating (305,336-342) each potential unit yield parameter includes:

determining (305) a current unit value of the yield parameter for the corresponding category in the future instance of the flight, and

calculating (336-342) each potential unit yield parameter as a sum of the corresponding historical unit yield parameter and current unit yield parameter weighted according to a corrective factor.

10. The yield management method (300) according to claim 9, wherein the step of calculating (336-342) the potential unit yield parameter further includes:

determining (336) a further first coefficient depending on a difference between the current time and a planned occurrence time of the future instance of the service,

determining (338) a further second coefficient depending on an increment of the at least one potential capacity variable with respect to the at least one current capacity variable for the category,

calculating (340) the corrective factor by combining the further  
5 first coefficient and the further second coefficient, and

calculating (342) the potential yield parameter as a sum of the historical yield parameter and the current yield parameter weighted according to the corrective factor.

10 11. The yield management method (300) according to any claim from 8 to 10, further including the steps of:

calculating (356,360) a weighted mean value of the potential yield parameter for each capacity variable of each category,

determining (355,358) a nesting order of the categories for each  
15 capacity variable according to the corresponding weighted mean potential yield parameters.

12. The yield management method (300) according to claim 11, wherein the step of determining each nesting order (355,358) includes:

20 providing (355) an input nesting order of the categories for each capacity variable, and

updating (358) each input nesting order by ranking the categories having at least one scenario with each component thereof that is strictly positive.

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13. The yield management method (300) according to any claim from 1 to 12, wherein the step of determining the authorisations (362-372) includes:

providing (362) an aggressiveness parameter indicative of an attitude to the risk,

30 defining (364) a first portion of a target function, the first portion calculating the yield parameter by assigning the offered capacity with a nesting policy,

defining (366,368) a second portion of the target function, the second portion calculating the yield parameter by assigning the offered  
35 capacity with an out of nesting policy,

defining (370) the target function as a sum of the first portion and the second portion weighted according to the aggressiveness parameter, and

calculating (372) the authorisations by optimising the target function.

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14. The yield management method (300) according to claim 13, wherein the

step of defining the second portion (366,368) includes defining an independent component of the second portion for each capacity variable.

15. The yield management method (300) according to any claim from 1 to 5 14, further including the step (348-354) of providing a user-defined scenario with a corresponding probability, the stochastic model being further applied to the user-defined scenario according to the corresponding probability.
- 10 16. The yield management method (300) according to any claim from 1 to 15, wherein the service consists of a cargo flight, the at least one capacity variable consists of a weight and a volume, and the yield parameter consists of a revenue.
- 15 17. A computer program application (225-245) directly loadable into a working memory (155) of a computer (110) for performing the method of any claims from 1 to 16 when the application program is run on the computer.
18. A program product (170) comprising a computer readable medium on 20 which the program application (2225-245) of claim 17 is stored.
19. A yield management system (100) for optimising a yield parameter resulting from assigning a capacity offered by a future instance of a service to each one of a plurality of different categories of requests 25 competing for the capacity, the capacity being defined by at least one capacity variable, the system including means (215) for storing a set of historical profiles for each one of a plurality of previous instances of the service, the set including a historical profile of an historical value of each capacity variable reserved by each category, means (225,235) for 30 assigning a probability to each previous instance of the service, means (220,230) for estimating a potential profile of a potential value of the capacity variable from each historical profile according to a corresponding current value of the capacity variable reserved by the category for the future instance of the service and according to a 35 corresponding unconstrained demand of the capacity variable for the category in the previous instance of the service, means (230) for defining a historical scenario for each previous instance of the service, the historical scenario including a final potential capacity variable from each corresponding potential profile, and means (240,245) for determining 40 an authorisation to allocate the offered capacity for each capacity variable of each category in the future instance of the flight by applying



a stochastic model to the historical scenarios according to the corresponding probabilities.

## ABSTRACT

## A YIELD MANAGEMENT METHOD AND SYSTEM

5       A yield management method (300) and system, particularly for  
maximising a revenue that can be obtained from a given capacity that is  
offered by a cargo flight; the capacity is defined by two different  
variables consisting of the weight and the volume of the freights that may  
be transported by the flight. A set of historical profiles of the capacity  
10 (weight and volume) reserved by each category is stored (306) for  
different previous instances of the flight. Corresponding potential  
profiles are estimated (316-326,332) independently for the weight and the  
volume. The estimation is carried out taking into consideration the  
corresponding capacity currently reserved by the category for a future  
15 instance of the flight; moreover, this result is emphasised according to a  
corresponding unconstrained demand for the category (not limited by the  
offered capacity). A series of historical scenarios of the demand at the  
departure of each previous instance of the flight are then obtained from  
the potential profiles (344). A stochastic model is applied (362-372) to  
20 the historical scenarios according to the corresponding probabilities for  
calculating the optimised weight and volume authorisations for each  
category.

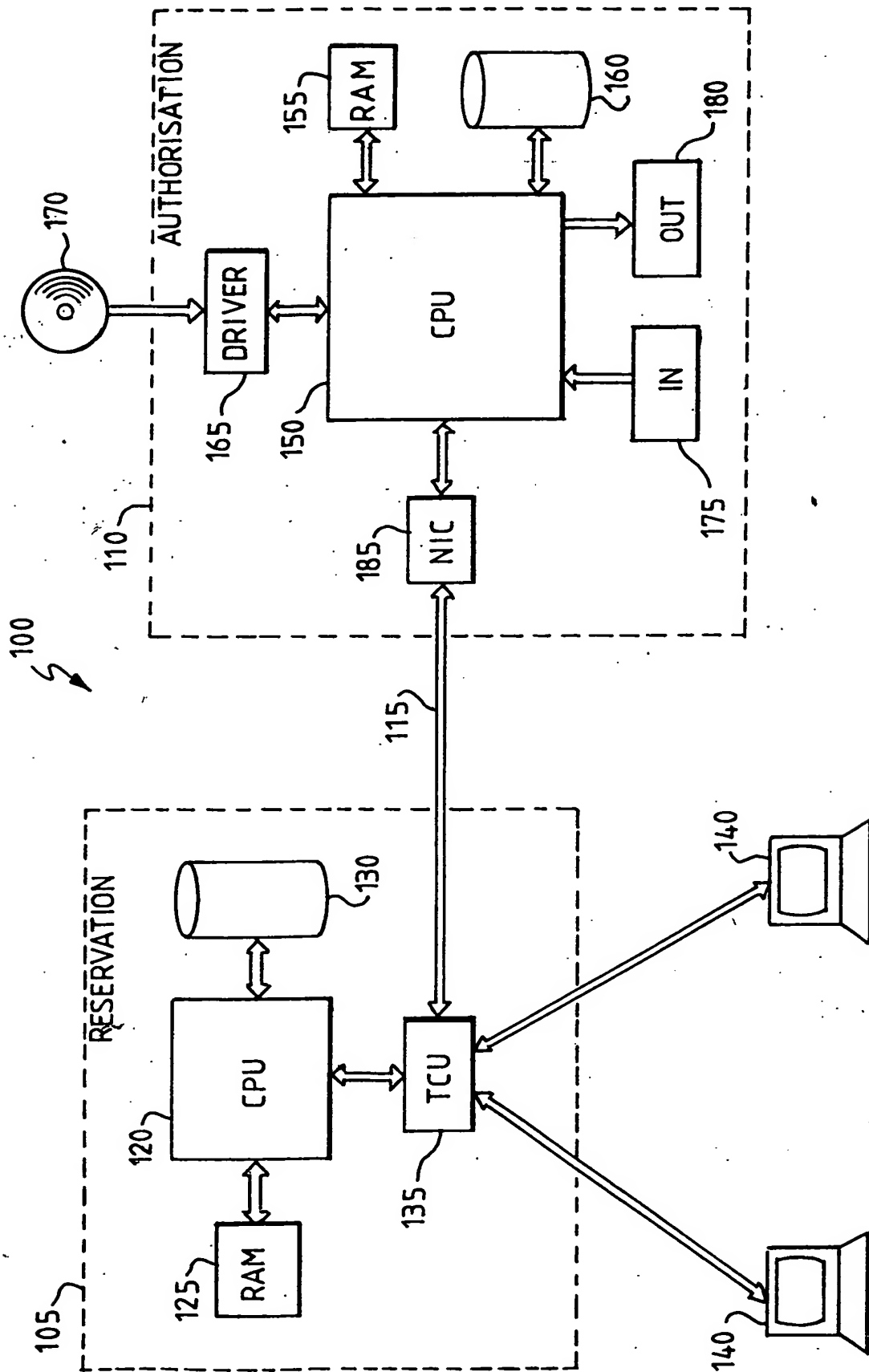


FIG. 1

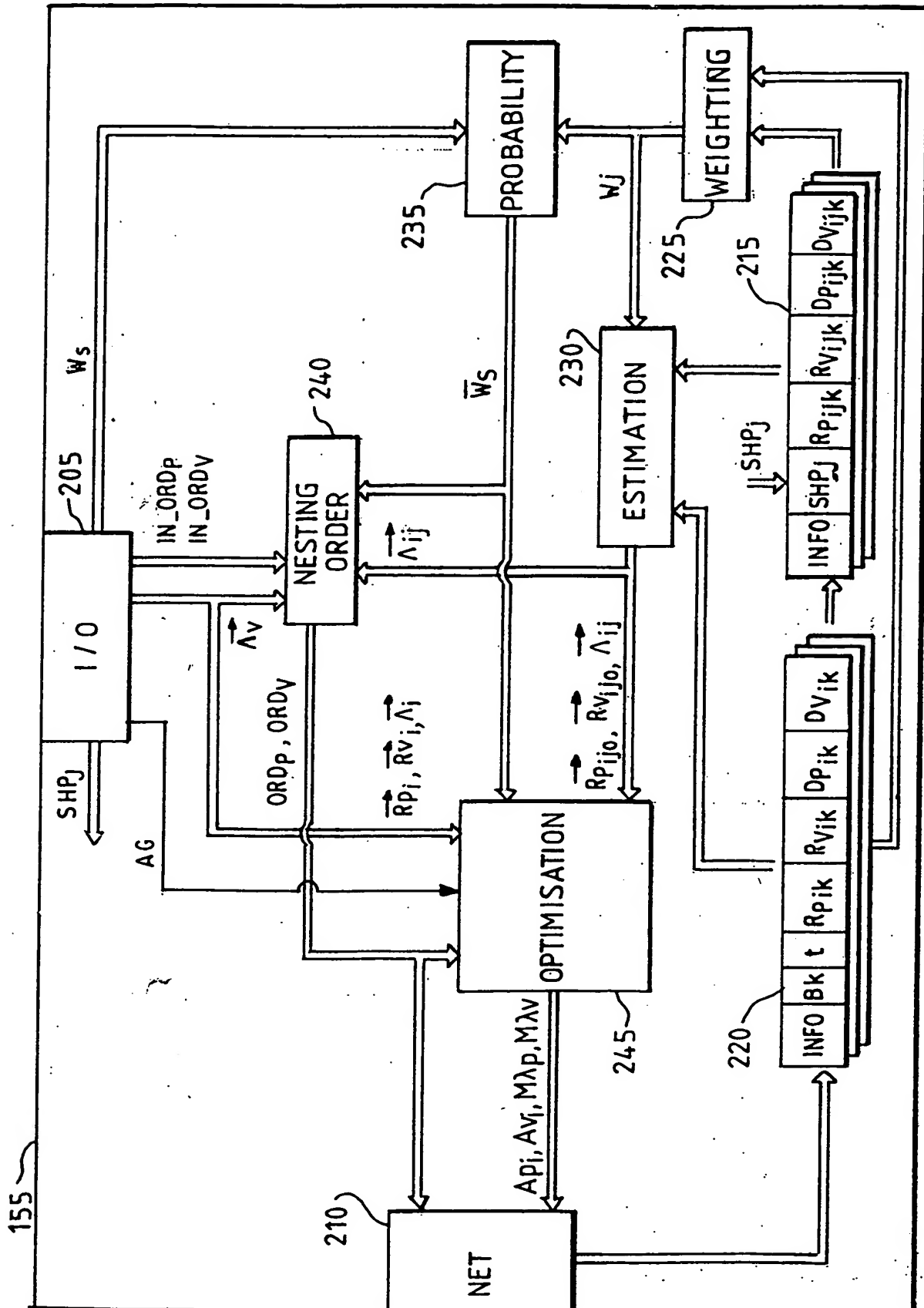
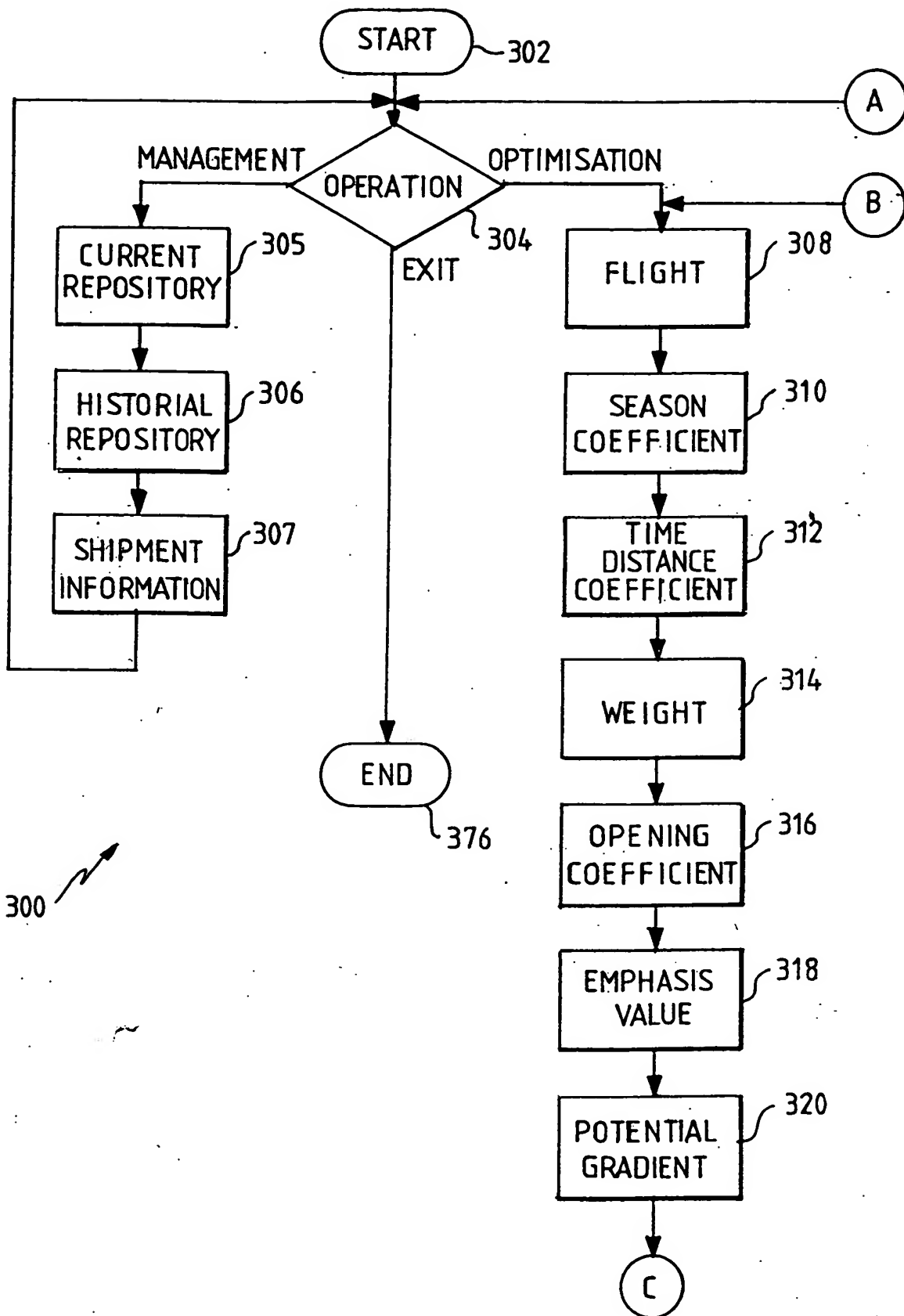


FIG. 2

**FIG. 3a**

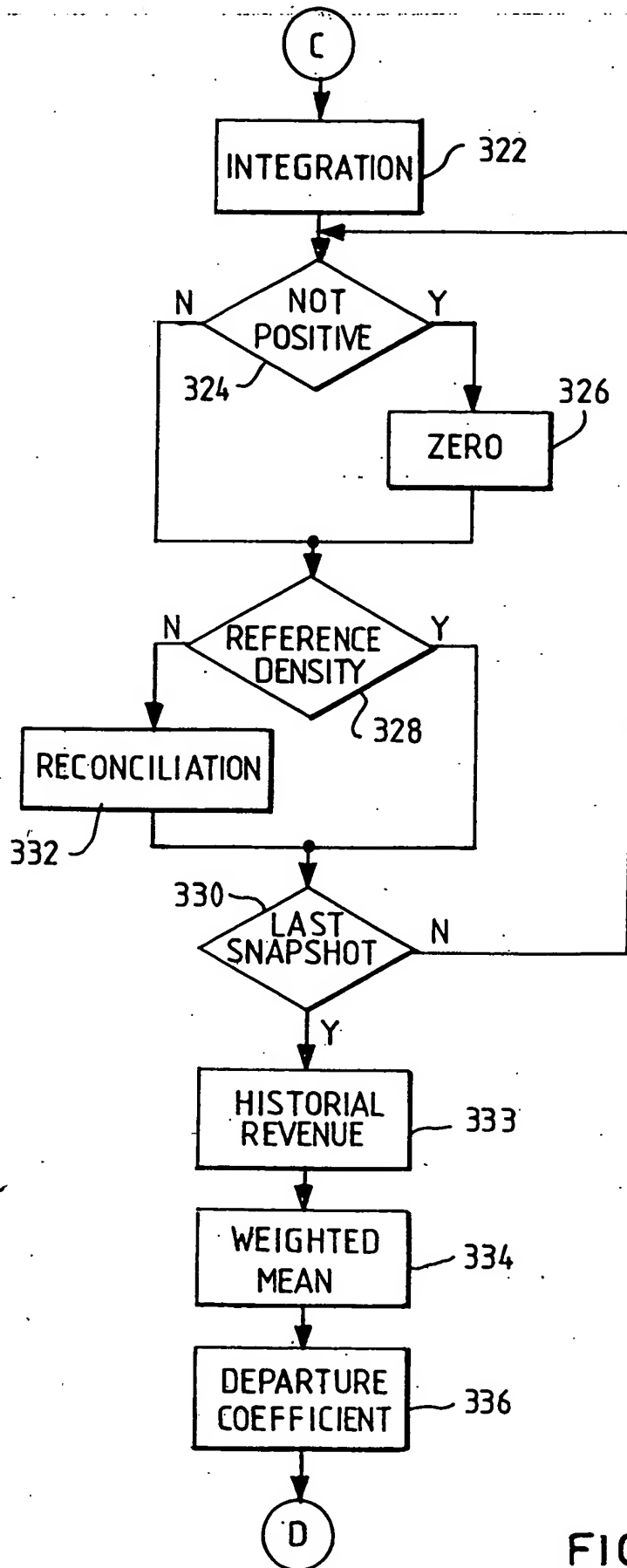


FIG. 3b

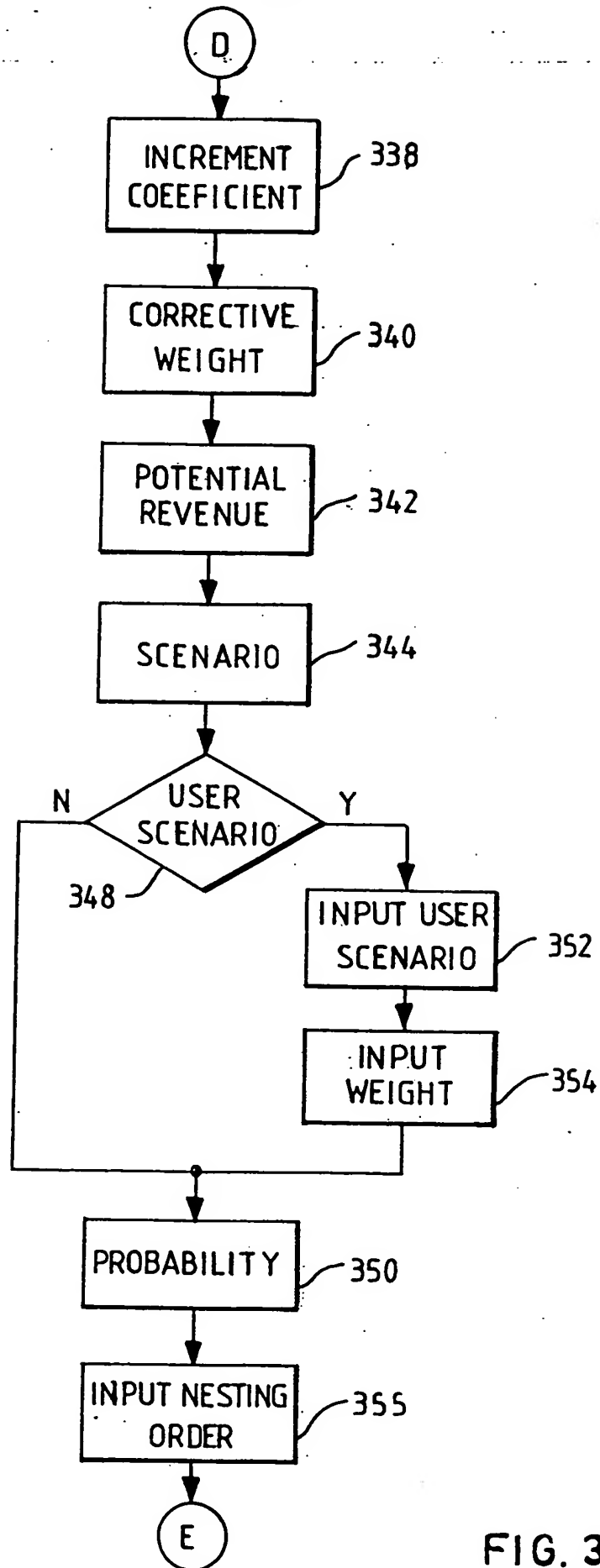


FIG. 3c

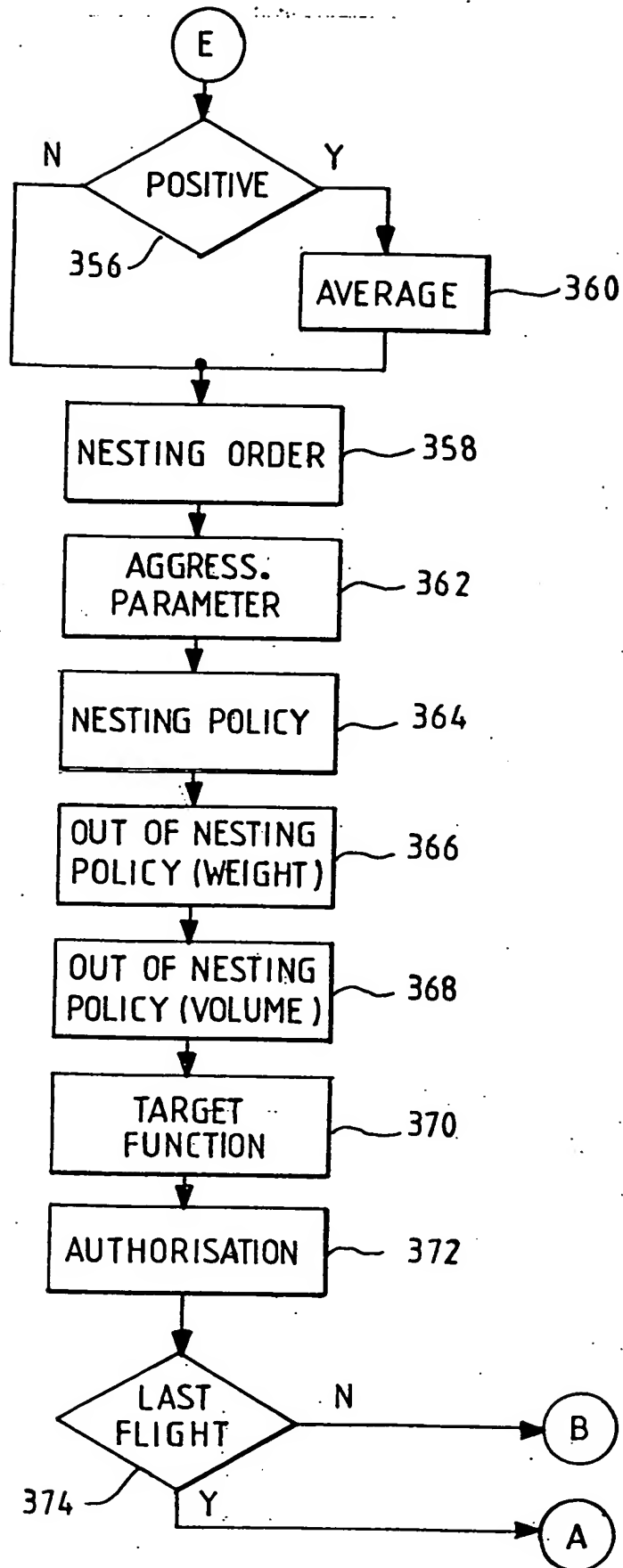
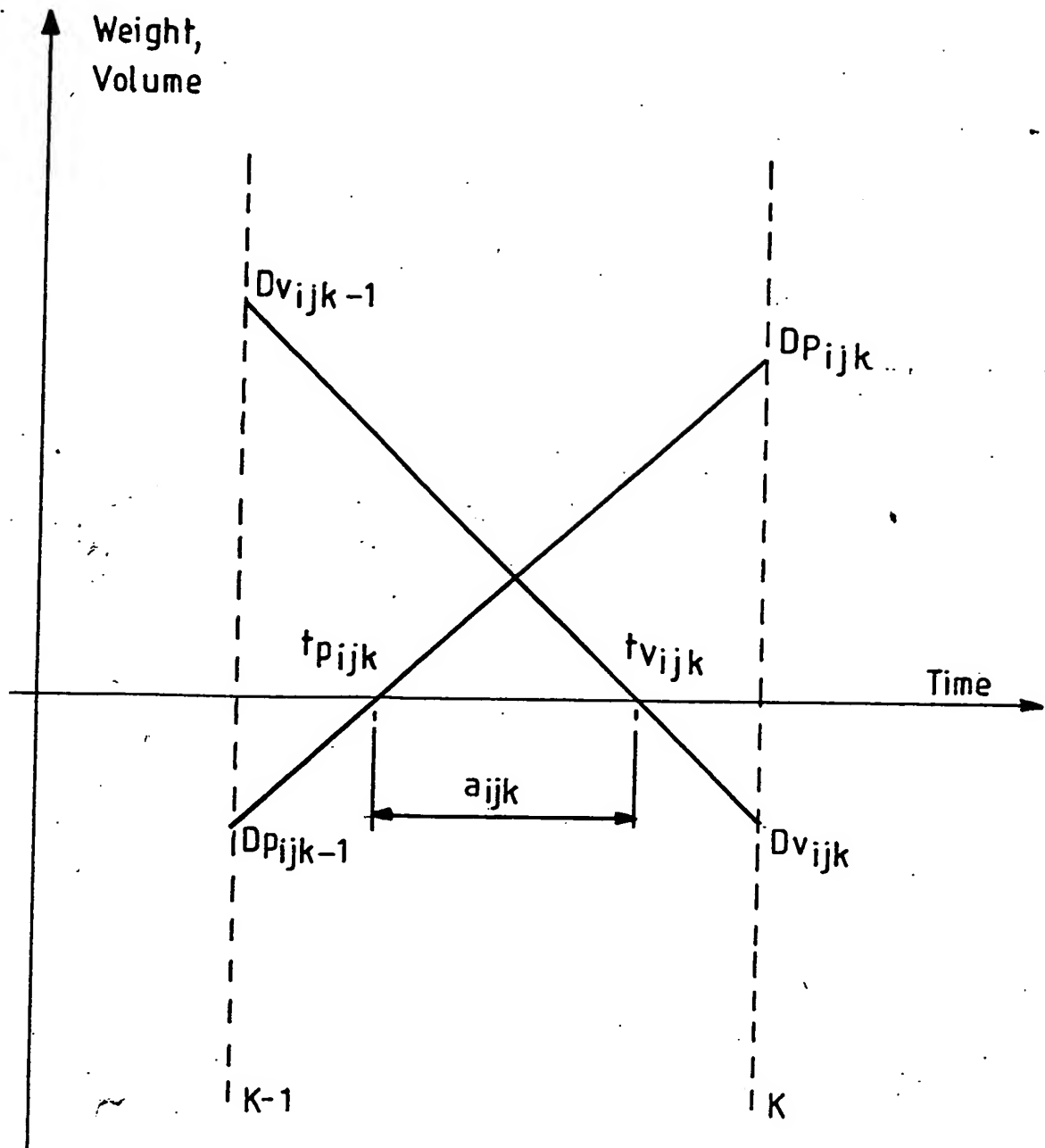
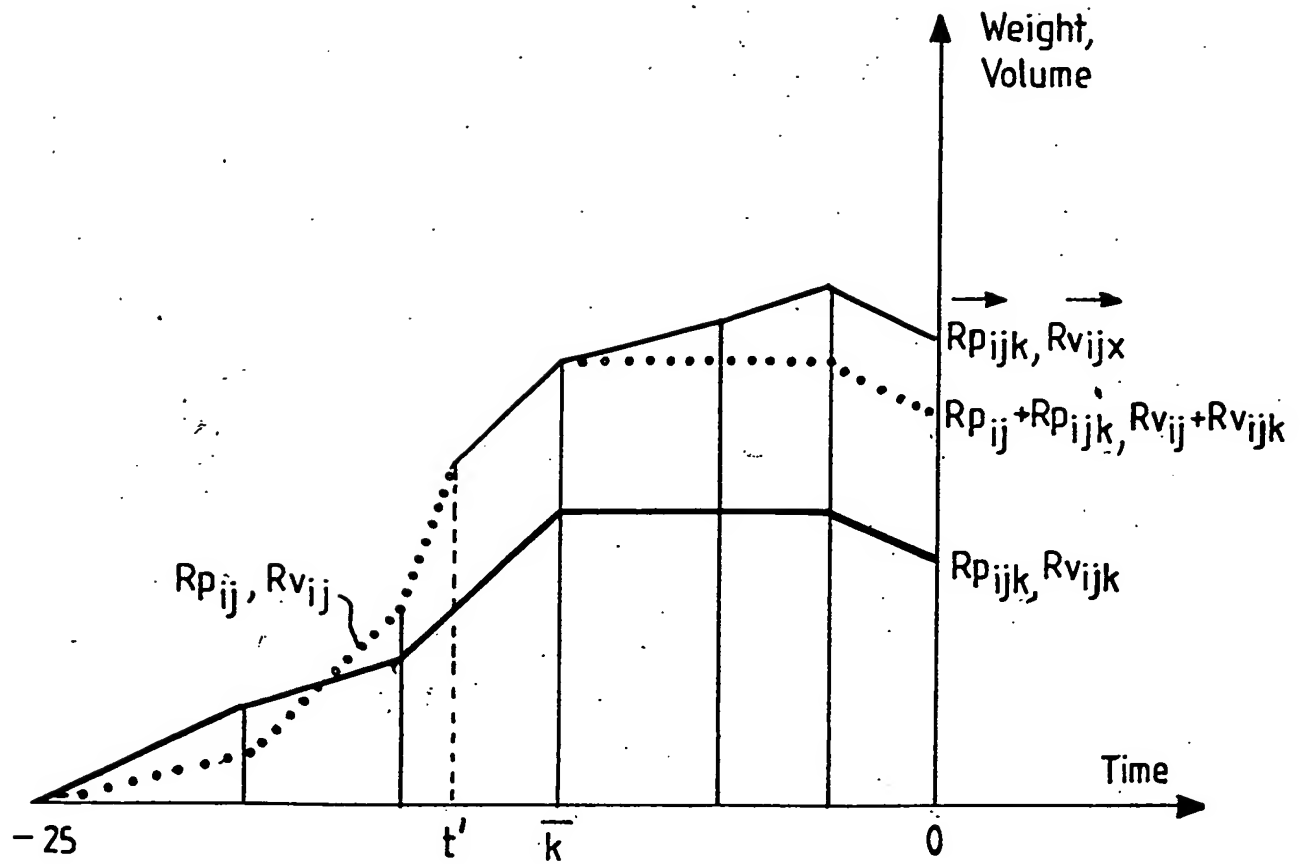


FIG. 3d



FIG 4a

FIG. 4b